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Core

Cambridge IGCSE®
Complete
Mathematics

Sixth Edition

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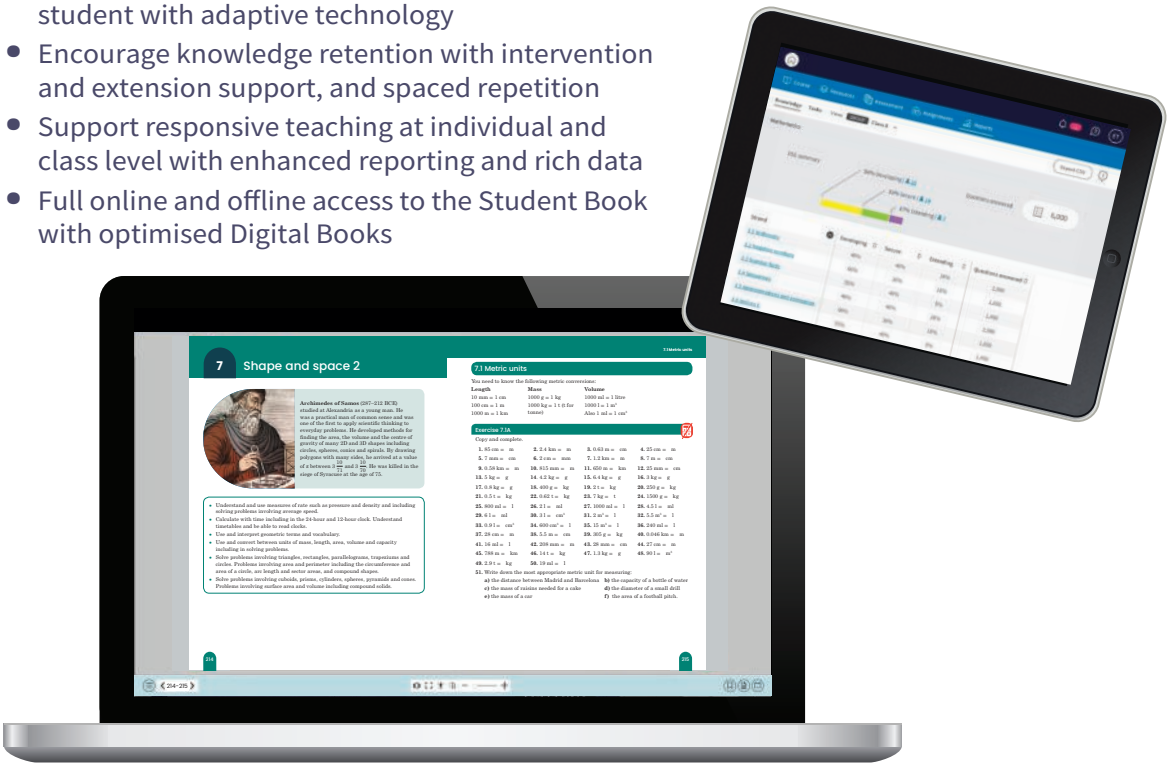

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Introduction

About this book

This book is designed specifically for the Cambridge IGCSE® Mathematics course. Experienced examiners have been involved in all aspects of the course, to ensure that the content adheres to the latest syllabus.

Using this book will ensure that you are well prepared for the exam at this level, and also studies beyond the IGCSE level in Mathematics. The features below are designed to make learning as interesting and effective as possible.

Finding your way around

To get the most out of this book when studying or revising, use the:

- **Contents list** to help you find the appropriate units
- **Index** to find key words so you can turn to any concept straight away.

Learning objectives

At the start of each chapter you will find a list of objectives. These will tell you what you should be able to do by the end of the chapter. They are based on what you need to cover for the Cambridge IGCSE syllabus.

Famous mathematicians

These are included at the start of each chapter to give you a brief insight into the life of a mathematician who played an important part in the development of the ideas contained in that chapter.

By finding out about the history of mathematics and considering a topic within the broader context of the subject, you can make connections between topics and develop a greater appreciation of how mathematics has developed over the centuries.

Worked examples

Worked examples are an important feature of the book and can be found in every sub-topic. These show you the important skills and techniques required in the exercises below and also provide a model for how to structure your solutions.

Exercises

There are thousands of questions in this book, providing ample opportunities to practise the skills and techniques required in the exam. The exercises contain questions of varying levels of difficulty, so that you can progress through a topic as your knowledge and confidence increases.

Each exercise has an icon to denote whether you can use a calculator or not. This  means you can use a calculator, while this  means you should not. The same icons also appear in the Revision Exercises.

Revision Exercise

At the end of each chapter, you will find revision questions to bring together all your knowledge and test your understanding of the contents of the chapter.

Examination-style questions

The revision exercises are followed by exam-style practice questions. These are very similar to the kind of questions you should expect to see in the real exam.

Tips

Yellow boxes throughout the exercises provide further information, hints on how to approach a question, or reminders of other concepts.

Answers

These can be found at the back of this book, so you can find out immediately whether or not you have answered a question correctly. Answers to all the numerical problems in the exercises, the review questions, and the exam-style questions are all included.

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Additional support can be found on Kerboodle. There are resources for every sub-topic, including adaptive assessments, personalised Next Steps and data-rich reports. You can also access the Digital Student Book.

2 Algebra 1

Isaac Newton (1643–1727) is thought by many to have been one of the greatest intellects of all time. He went to Trinity College Cambridge in 1661 and by the age of 23 he had made three major discoveries: the nature of colours, calculus and the law of gravitation. He used his version of calculus to give the first satisfactory explanation of the motion of the Sun, the Moon and the stars. Because he was extremely sensitive to criticism, Newton was always very secretive, but he was eventually persuaded to publish his discoveries in 1687.

- Substitute into expressions and formulae.
- Simplify expressions, expand brackets and factorise.
- Construct and solve linear equations including those where x appears in the denominator as part of a linear expression.
- Solve simultaneous equations.
- Understand and use rules of indices including positive, zero and negative.
- Understand and use linear inequalities including representing on a number line.
- Understand sequences including continuing a sequence, recognising patterns, term-to-term rules, finding and using nth terms and relationships between sequences.

2.1 Introduction to algebra

Algebra is when you use letters to stand for generalised numbers.

3, 7, 8.3 and −4.61 are all numbers, whereas a letter can be used to stand for a general number.

The letter may take a single value, two possible values, or many more possible values, depending on the context in which it is used.

Algebra is made up of building blocks, called **terms**. A term is a single algebraic object.

These are examples of terms: $2a$ x^2 $3pq$ $\frac{x}{w}$

There are some conventions that you need to know when it comes to writing terms.

- If there is no number in front of the letter, for example x, this means $1 \times x$.
- If two letters are side by side, it means they are multiplied together, for example ab means $a \times b$.
- If a letter is squared, it means the letter multiplied by itself, for example p^2 means $p \times p$.

When terms are added or subtracted, they make **expressions**.

These are examples of expressions: $x + y$ $a^2 - 2$ $pqr + 3s - 5t$

The number in front of a letter (or letters) is called the **coefficient**.

The coefficient of x in the expression $4x + y$ is 4.

If a number appears on its own in an expression, it is called a **constant term**.

The constant term in the expression $5y + 7$ is 7.

Exercise 2.1A

1. Consider this expression: $x + 3y$

a) How many terms are there in the expression?

b) What is the coefficient of

i) y

ii) x?

2. Consider this expression: $9x - 5y + 6z + 7$

a) How many terms are there in the expression?

b) What is the coefficient of

i) x

ii) y?

c) What is the value of the constant term?

3. Consider this expression: $2a + 7b - 3ab + 4c - 6$

a) How many terms are there in the expression?

b) What is the coefficient of

i) b

ii) ab?

c) What is the value of the constant term?

4. Bethia is considering the expression $x - 7y + 3$.

She says, 'There are two terms in the expression since there are two letters. The coefficient of x is 0 since there is no number in front of it and the coefficient of y is 7 since there is a 7 in front of it'.

Is Bethia correct? Explain your answer.

5. Maya has x counters. Jaya has 3 more counters than Maya. Keira has four times as many counters as Maya.

a) Write down an expression for:

i) the number of counters that Jaya has

ii) the number of counters that Kiera has.

b) Maya gives 8 counters to her brother.

Write down an expression for the number of counters that Maya has now.

6. Amaar has a pears. Bharat has b pears. Chico has five times as many pears as Bharat.

Write down an expression for:

a) the number of pears that Amaar and Bharat have

b) the number of pears that Amaar and Chico have

c) the number of pears that Bharat and Chico have.

7. A square has side length p cm.

Write down an expression for:

a) the area of the square

b) the perimeter of the square.

8. A rectangle has length x cm and width y cm.

Write down an expression for:

a) the area of the rectangle

b) the perimeter of the rectangle.

When you have lots of terms in an expression, some of them may be of the same type.

In the expression $4x + 3y + 2x$, there are two terms in x: the $4x$ term and the $2x$ term. These are called **like terms**.

You can collect like terms by adding (or subtracting) them to simplify an expression:

$4x + 3y + 2x = 4x + 2x + 3y = 6x + 3y$

Example

Simplify:

a) $4x + 13y + 5x - 2y - x$

b) $3x^2 + 5x + 6x^2 - 4x - 2x^2$

a) $4x + 5x - x + 13y - 2y$ Rewrite the terms so that like terms are together.

$= 8x + 11y$ Then add or subtract the like terms.

b) $3x^2 + 6x^2 - 2x^2 + 5x - 4x$ Rewrite the order of the terms. Note that terms in x^2 and terms in x are *not* like terms.

$= 7x^2 + x$ Add or subtract the like terms.

Exercise 2.1B

Simplify these expressions.

1. $3x + 2y + 4x$ **2.** $2x + 7y + 3x - 4y$ **3.** $8a + 4b - 6a - 7b$

4. $2p + 3q - 7p - q$ **5.** $4x^2 + 6x + 2x^2 - 11x$ **6.** $5a^2 - 3a - 2a^2 + 9a$

7. $xy + 3x + 2y + 4xy$ **8.** $2p^2 + 2q^2 + 4pq - p^2 - 5q^2$ **9.** $3c + 4d + 5e - 6c - 7d - e$

10. $14a - 11b + 3ab + 2a^2 - 7b - 3a + 4b^2 - 9ab$

You can also substitute numbers into an expression to work out the value.

Example

Find the value of $2x + 4y$ when:

a) $x = 4$, $y = 7$

b) $x = 3$, $y = -5$

a) $2 \times 4 + 4 \times 7$
$= 8 + 28$
$= 36$

b) $2 \times 3 + 4 \times (-5)$
$= 6 - 20$
$= -14$

Replace the letters with the numbers and evaluate. Remember to use BIDMAS.

Tip

Take care when substituting negative numbers into expressions.

Exercise 2.1C

If $a = -4$, $b = 5$, $c = -2$, work out:

1. $2a + 3$	**2.** $3b - 7$	**3.** $4a - 1$	**4.** $2b + c$
5. $5c - 2a$	**6.** $6a - 3$	**7.** $2c + b$	**8.** $3a - 2b$
9. $6c - 2b$	**10.** $3c + 4a$	**11.** $3c - 4$	**12.** $2a - 3c$
13. $7b + 3a$	**14.** $8a + 6c$	**15.** $2b - 4a$	**16.** $4b + 5$
17. $3a + 8$	**18.** $2c - a$	**19.** $5a - 2c$	**20.** $3b + 7$

If $n = 3$, $x = -1$, $y = 6$, work out:

21. $2x - 3$	**22.** $3y + 4n$	**23.** $5n + 2x$	**24.** $4y - x$
25. $7y - 2$	**26.** $3x + 2n$	**27.** $10x + 5$	**28.** $6x - y$
29. $4x - 5y$	**30.** $2y - 10$	**31.** $8n - 2y$	**32.** $7n + 3y$
33. $6y + 4$	**34.** $4n + 5x$	**35.** $2n + 3x$	**36.** $5y - 20$
37. $9y - n$	**38.** $8x + 2n$	**39.** $5x + 6$	**40.** $3n - 2x$

2.2 Sequences

A sequence is a list of numbers that follow a pattern.

Each number in the sequence is called a **term**.

How you get from one term in the sequence to the next is called the **term-to-term rule**.

Consider the sequence 4, 6, 8, 10, …

The term-to-term rule is 'add 2'.

Consider the sequence 5, 10, 20, 40, …

The term-to-term rule is '× 2'.

Exercise 2.2A

1. Find the next number in each sequence and write down the term-to-term rule.

a) 1, 5, 9, 13, …

b) 39, 36, 33, 30, …

c) 3, 6, 12, 24, …

d) 4, 9, 15, 22, …

e) 200, 100, 50, 25, …

f) 88, 99, 110, …

2. Write down each sequence, find the missing number and write down the term-to-term rule.

a) 1, 6, □, 16

b) 1, 2, 4, 8, □

c) □, 2, 5, 8, 11

d) 2400, 240, 24, □

e) 1, 2, 4, 7, □, 16

f) 12, 8, 4, □, −4

3. Here is the start of a sequence: 1, 3, 4, …

Each new term is found by adding the previous two terms. For example, 4 = 1 + 3

The next term will be 7.

a) Write down the next six terms.

b) Use the same rule to write down the next four terms of the sequence which starts 2, 5, 7, …

c) The Fibonacci sequence is a special case of this type of sequence.

The Fibonacci sequence starts 1, 1, …

Write down the first ten terms of the Fibonacci sequence.

4. a) Write down the next two lines of the sequence:

$3 \times 4 = 3 + 3^2$

$4 \times 5 = 4 + 4^2$

$5 \times 6 = 5 + 5^2$

b) Complete the lines below:

$10 \times 11 =$

$30 \times 31 =$

5. Copy and complete the pattern and write down the next two lines.

$1 + 9 \times 0 = 1$

$2 + 9 \times 1 = 11$

$3 + 9 \times 12 = 111$

$4 + 9 \times 123 = 1111$

$5 + 9 \times 1234 =$

6. For the sequence 2, 3, 8, … each new term is found by squaring the previous term and then subtracting 1. Write down the next two terms.

7. The sequence 3, 3, 5, 4, 4 is obtained by counting the letters in 'one, two, three, four, five, …'. Write down the next three terms.

8. The odd numbers 1, 3, 5, 7, 9, … can be added to give an interesting sequence.

1	=	1	=	$1 \times 1 \times 1$
$3 + 5$	=	8	=	$2 \times 2 \times 2$
$7 + 9 + 11$	=	27	=	$3 \times 3 \times 3$
$13 + 15 + 17 + 19$	=	64	=	$4 \times 4 \times 4$

1, 8, 27, 64 are **cube** numbers.

You write $2^3 = 8$ ['two cubed equals eight']

$4^3 = 64$

Continue adding the odd numbers in the same way as shown above. Do you *always* get a cube number?

9. a) Write down the next three lines of this pattern.

$$1^3 = 1^2 = 1$$
$$1^3 + 2^3 = (1 + 2)^2 = 9$$
$$1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2 = 36$$

b) Work out as simply as possible:

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$

10. Here are the sequences of the first six odd and the first six even numbers.

	1st	2nd	3rd	4th	5th	6th
odd	1	3	5	7	9	11
even	2	4	6	8	10	12

Find: **a)** the 8th even number **b)** the 8th odd number

c) the 13th even number **d)** the 13th odd number.

e) Explain in words the relationship between the nth odd number and the nth even number.

f) If the 57th even number is 114, what is the 57th odd number?

g) Write down:

i) the 45th even number **ii)** the 53rd odd number

iii) the 100th odd number **iv)** the 219th odd number.

11. Here the numbers 1 to 21 have been written in three columns.

			Line
1	2	3	**1**
4	5	6	**2**
7	8	9	**3**
10	11	12	**4**
13	14	15	**5**
16	17	18	**6**
19	20	21	**7**

left middle right

a) What number will you get on the right of:

i) line 8 **ii)** line 12 **iii)** line 25?

b) Write down the number in the middle of:

i) line 8 **ii)** line 12 **iii)** line 20.

c) What number will you get in:

i) line 10 on the left

ii) line 13 on the right

iii) line 17 in the middle

iv) line 30 on the left?

d) Find the missing number:

i) 120 is on the right of line _____.

ii) 61 is on the left of line _____.

iii) 92 is in the middle of line _____.

iv) 148 is on the left of line _____.

12. Here is a sequence:

1, 3, 6, 10, 15, …

a) Find the difference between each pair of successive terms of the sequence.

b) Describe the way in which the differences go up.

c) Use your answers to parts **(a)** and **(b)** to find:

i) the 8th term

ii) the 12th term

iii) the 15th term.

2.3 Finding a rule

For the sequence 3, 8, 13, 18, … the term-to-term rule is ‘add 5’. You can draw a mapping diagram with a column for 5 times the term number (i.e. $5n$).

n	$5n$	term
1	5	$\xrightarrow{-2}$ 3
2	10	$\xrightarrow{-2}$ 8
3	15	$\xrightarrow{-2}$ 13
4	20	$\xrightarrow{-2}$ 18
n	$5n$	$\xrightarrow{-2}$ $5n - 2$

You can see that each term is 2 less than $5n$.

So, the 10th term is $(5 \times 10) - 2 = 48$

and the 20th term is $(5 \times 20) - 2 = 98$

The nth term is $5 \times n - 2 = 5n - 2$

Sometimes the terms of the sequence decrease.

For the sequence 8, 6, 4, 2, … the term-to-term rule is 'subtract 2'.

You can draw a mapping diagram with a column for -2 times the term number (i.e. $-2n$).

n	$-2n$	term
1	-2	$\xrightarrow{+10}$ 8
2	-4	$\xrightarrow{+10}$ 6
3	-6	$\xrightarrow{+10}$ 4
4	-8	$\xrightarrow{+10}$ 2
n	$-2n$	$\xrightarrow{+10}$ $10 - 2n$

You can see that each term is 10 more than $-2n$.

So, the 5th term is $(-2 \times 5) + 10 = 0$

and the 9th term is $(-2 \times 9) + 10 = -8$

The nth term is $-2n + 10 = 10 - 2n$

In general, the number that n is multiplied by is the number in the term-to-term rule.

The nth term is called the **position-to-term** rule and means that you can find, for example, the 100th term in the sequence without having to work out all 100 terms.

For the first sequence above, with nth term $5n - 2$, the 100th term is $5 \times 100 - 2 = 498$.

For the second sequence above, with nth term $10 - 2n$, the 100th term is $10 - 2 \times 100 = -190$.

Exercise 2.3A

1. Look at the sequence 5, 8, 11, 14, …

The difference between terms is 3.

Copy and complete: 'The nth term of the sequence is $3n + \ldots$'.

n	$3n$	term
1	3	$\xrightarrow{+\square}$ 5
2	6	8
3	9	11
4	12	14

2. Look at the sequences and tables below. Find the nth term in each case.

a) Sequence 5, 9, 13, 17, …

n	$4n$	term
1	4	5
2	8	9
3	12	13
4	16	17

nth term =

b) Sequence 2, 8, 14, 20, …

n	$6n$	term
1	6	2
2	12	8
3	18	14
4	24	20

nth term =

3. In the sequence 6, 11, 16, 21, …

the difference between terms is 5.
Copy and complete the table and write an expression for the nth term of the sequence.

n		term
1		6
2		11
3		16
4		21
10		

4. Look at the sequence 6, 10, 14, 18, …

a) Write down the difference between terms. Make a table like the one in Question **3** and use it to find an expression for the nth term.

b) Use your nth term to find the 20th term of the sequence.

5. Write down each sequence in a table. Find the nth term and the 50th term of each sequence.

a) 5, 7, 9, 11, …

b) 3, 7, 11, 15, …

c) 2, 8, 14, 20, …

In Questions **6** to **17** you are given a sequence in a table. Copy the table and make an extra column. Find an expression for the nth term of each sequence. [t stands for 'term'.]

6.

n	t
1	3
2	8
3	13
4	18
5	23

7.

n	t
1	15
2	12
3	9
4	6
5	3

8.

n	t
1	7
2	13
3	19
4	25
5	31

9.

n	t
1	14
2	12
3	10
4	8
5	6

10.

n	t
1	6
2	7
3	8
4	9
5	10

11.

n	t
1	13
2	10
3	7
4	4
5	1

12.

n	t
1	5
2	13
3	21
4	29
5	37

13.

n	t
1	27
2	22
3	17
4	12
5	7

14.

n	t
6	26
7	23
8	20
9	17
10	14

15.

n	t
8	83
9	93
10	103
11	113
12	123

16.

n	t
4	15
5	18
6	21
7	24

17.

n	t
12	76
13	71
14	66
15	61

18. Make a table for each sequence. Find the nth term and the 20th term of each sequence.

a) 26, 18, 10, 2, …

b) 16, 13, 10, 7, …

c) 48, 39, 30, 21, …

19. The second term of a linear sequence is 16 and the fourth term is 30.

Find the 10th term of the sequence.

20. Hari says that 53 is a term in the sequence with nth term $6n + 2$.

Explain why Hari is incorrect.

21. a) Write down the nth term of the sequence 2, 4, 6, 8, …

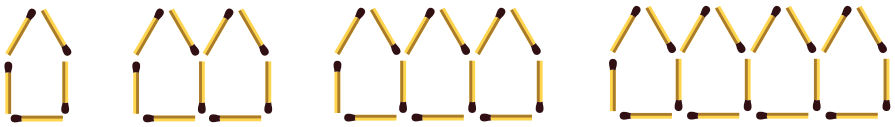
b) Write down the nth term of the sequence 1, 3, 5, 7, …

c) Use your answers to **(a)** and **(b)** to explain why the sum of an odd number and an even number is always odd.

Tip

A **linear sequence** is a sequence where the difference between one term and the next is constant.

Here is a sequence of 'houses' made from matches.



The table on the right records the number of houses h and the number of matches m.

h	m
1	5
2	9
3	13
4	17

Since the term-to-term rule is 'add 4' the sequence is based on $4h$.

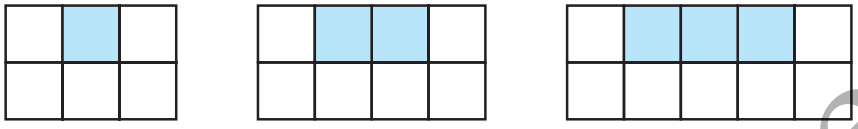
Now you can see that m is one more than $4h$.

So the formula linking m and h is: $m = 4h + 1$

h	$4h$	m
1	4	5
2	8	9
3	12	13
4	16	27

Exercise 2.3B

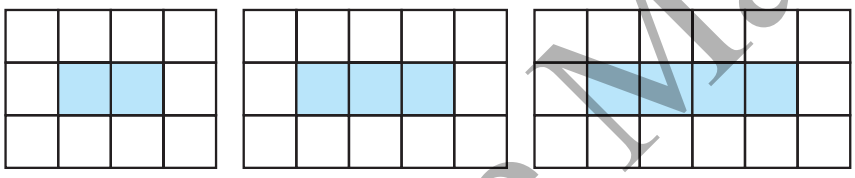
1. Below is a sequence of diagrams showing blue tiles (b) and white tiles (w) with the related table.



What is the formula for w in terms of b? [i.e. write '$w = \ldots$']

b	w
1	5
2	6
3	7
4	8

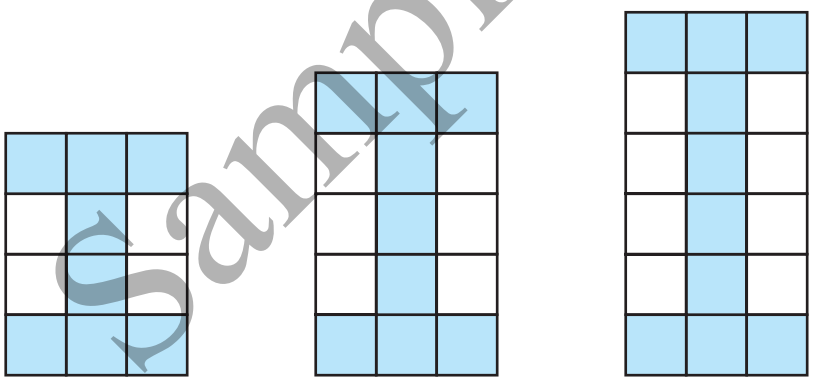
2. This is a different sequence with blue tiles (b) and white tiles (w) and the related table.



What is the formula that links b and w? Write it as '$w = \ldots$'.

b	w
2	10
3	12
4	14
5	16

3. Here is a sequence of diagrams made from square tiles.



Make your own table for blue tiles (b) and white tiles (w).
What is the formula for w in terms of b?

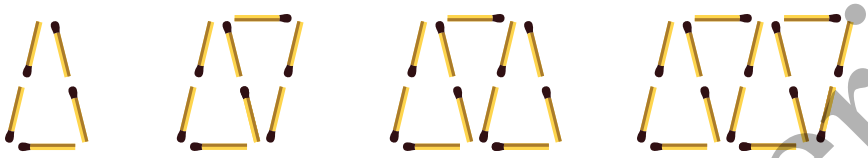
4. In this sequence, the number of matches is m and the number of triangles is t.

Make a table for t and m. It starts like the table on the right:

t	m
1	3
2	5
.	.
.	.
.	.

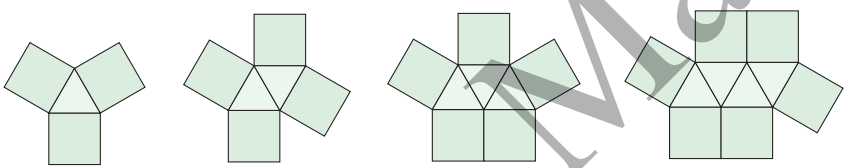
Continue the table and find a formula for m in terms of t. Write '$m = \ldots$'.

5. Here is a different sequence of matches and triangles.



Make a table and find a formula connecting m and t.

6. In this sequence there are triangles (t) and squares (s) around the outside.



What is the formula connecting t and s?

7. Look at the tables below. In each case, find a formula connecting the two letters.

a)

n	p
1	3
2	8
3	13
4	18

write '$p = \ldots$'

b)

n	k
2	17
3	24
4	31
5	38

write '$k = \ldots$'

c)

n	w
3	17
4	19
5	21
6	23

write '$w = \ldots$'

8. In these tables it is harder to find the formula because the numbers on the left do not go up by one each time. Try to find a formula in each case.

a)

n	y
1	4
3	10
7	22
8	25

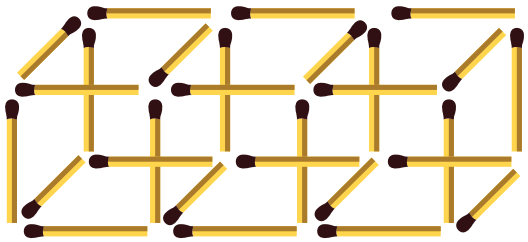
b)

n	h
2	5
3	9
6	21
10	37

c)

n	w
3	14
7	26
9	32
12	41

9. This is one shape in a sequence of cubes made from matches.



Find a formula connecting the number of matches (m) and the number of cubes (c).

Some sequences are not linear.

You already know the sequence of square numbers 1, 4, 9, 16, … . The nth term is n^2.

You already know the sequence of cube numbers 1, 8, 27, 64, … . The nth term is n^3.

You can use these sequences to find the nth term of other sequences.

The terms in the sequence 3, 6, 11, 18, … are all two more than the terms in the sequence of square numbers, so the nth term is $n^2 + 2$.

Sequences that are based on the square numbers are called **quadratic sequences**.

The terms in the sequence 0, 7, 26, 63, … are all one less than the terms in the sequence of cube numbers, so the nth term is $n^3 - 1$.

Sequences that are based on the cube numbers are called **cubic sequences**.

Exercise 2.3C

Find the first five terms of each of these sequences.

1. $n^2 + 4$ **2.** $n^2 + n$ **3.** $n^3 + 2$ **4.** $n^3 - n$

For each of these sequences, find an expression for the nth term and find the 20th term.

5. 4, 7, 12, 19, …

6. 2, 8, 18, 32, …

7. 0, 3, 8, 15, …

8. 0.5, 2, 4.5, 8, …

9. −6, −3, 2, 9, …

10. −1, −4, −9, −16, …

11. 0, −3, −8, −15, …

12. 2, 9, 28, 65, …

13. 2, 16, 54, 128, …

14. −1, 6, 25, 62, …

Tip

Write out the sequence of **cube** numbers and compare.

2.4 Brackets and factors

a) Expand $2(3x + 2)$

$2(3x + 2) = 2 \times 3x + 2 \times 2$
$= 6x + 4$

Multiply each term in the bracket by the number in front.

b) Expand $2x(3 + 4x)$

$2x(3 + 4x) = 2x \times 3 + 2x \times 4x$
$= 6x + 8x^2$

Multiply each term in the bracket by the term in front.

c) Expand and simplify $3(2x + 5) + 4(x - 2)$

$3(2x + 5) + 4(x - 2) = 6x + 15 + 4x - 8$
$= 10x + 7$

Expand each bracket first.
Collect like terms.

Exercise 2.4A

Expand the brackets.

1. $3(x + 3)$ **2.** $4(x - 2)$ **3.** $5(2x + 1)$

4. $4(a + 7)$ **5.** $6(2x + 1)$ **6.** $10(5 - x)$

7. $3(4x + 5)$ **8.** $9(3 + x)$ **9.** $5(y - 2)$

10. $7(a - 2)$ **11.** $11(2x - y)$ **12.** $8(3x + 2y)$

Expand the brackets.

13. $x(4 + 2x)$ **14.** $x(3x - 2)$ **15.** $y(4y + 7)$

16. $a(2a - 6)$ **17.** $3x(4 - 6x)$ **18.** $4y(3y + 5)$

19. $5a(3a - 4)$ **20.** $7p(6 - 7p)$ **21.** $3x(4x + x^2)$

22. $4x(x^2 - 3x)$

Expand and simplify.

23. $3(4x + 1) + 4(2x - 1)$ **24.** $2(6x - 1) + 5(x - 3)$

25. $x(2x + 3) + x(4x + 1)$ **26.** $5(2x + 7) - 2(x + 4)$

27. $6(3x + 2) - 4(2x - 1)$ **28.** $2x(x + 3) - 3x(x - 4)$

Tip

Take care when expanding with negative numbers. Remember that multiplying a negative by a positive gives a negative answer and multiplying two negatives gives a positive answer.

Example

a) Expand $(x + 3)(x + 2)$

Method 1: Using a grid

	x	$+3$
x	x^2	$+3x$
$+2$	$+2x$	$+6$

So $(x + 3)(x + 2) = x^2 + 3x + 2x + 6$

$= x^2 + 5x + 6$

Method 2: Using **FOIL**

$(x + 3)(x + 2) = x \times x + 2 \times x + 3 \times x + 3 \times 2$

$= x^2 + 2x + 3x + 6$ Add like terms.

$= x^2 + 5x + 6$

b) Expand $(x - 4)(x + 7)$

Method 1: Using a grid

	x	-4
x	x^2	$-4x$
$+7$	$+7x$	-28

So $(x - 4)(x + 7) = x^2 - 4x + 7x - 28$ Add like terms.

$= x^2 + 3x - 28$

Method 2: Using **FOIL**

$(x - 4)(x + 7) = x \times x + 7 \times x - 4 \times x - 4 \times 7$

$= x^2 + 7x - 4x - 28$ Add like terms.

$= x^2 + 3x - 28$

Tip

FOIL stands for **F**irst, **O**utside, **I**nside, **L**ast.

Multiply the **F**irst terms in each bracket.

Multiply the **O**utside terms in each bracket.

Multiply the **I**nside terms in each bracket.

Multiply the **L**ast terms in each bracket.

Revision exercise 2

1. Write down each sequence, find the next two numbers and write down the term-to-term rule.

a) 2, 9, 16, 23, … **b)** 20, 18, 16, 14, …

c) −5, −2, 1, 4, … **d)** 128, 64, 32, 16, …

e) 8, 11, 15, 20, …

2. Look at the number pattern below.

$(2 \times 1) - 1 = 2 - 1$

$(3 \times 3) - 2 = 8 - 1$

$(4 \times 5) - 3 = 18 - 1$

$(5 \times 7) - 4 = 32 - 1$

$(6 \times a) - 5 = b - 1$

a) What number does the letter a stand for?

b) What number does the letter b stand for?

c) Write down the next line in the pattern.

3. $1 + 3 = 2^2$

$1 + 3 + 5 = 3^2$

a) $1 + 3 + 5 + 7 = x^2$

Calculate x.

b) $1 + 3 + 5 + \ldots + n = 100$

Calculate n.

4. Here is a sequence:

$$1 + 2 + 1 = 2^2$$

$$1 + 2 + 3 + 2 + 1 = 3^2$$

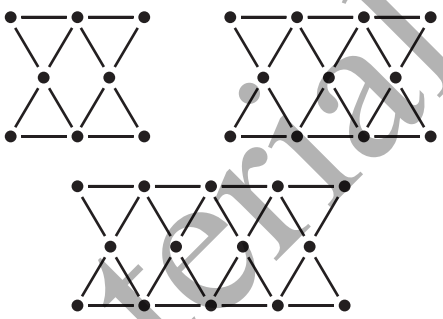
$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 4^2$$

a) Write down the next two lines of the sequence.

b) Complete the line below:

$1 + 2 + 3 + \ldots + 1 = 9^2$

5. Here are three diagrams with lines and dots.



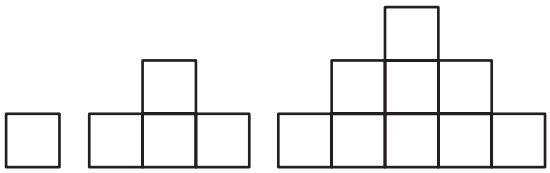
a) Find a formula connecting the number of lines l and the number of dots d.

b) How many dots are there in a diagram with 294 lines?

c) Josh says that a pattern in this sequence will have 401 lines.

Is Josh correct? Explain your answer.

6.



a) Draw the next diagram in this sequence.

b) Write down the number of squares in each diagram.

c) Describe in words the sequence you obtained in part **(b)**.

d) How many squares will there be in the diagram that has 13 squares on the base?

7. Each diagram in the sequence below consists of a number of dots.

Diagram number	1	2	3
Diagram			

a) Draw diagram number 4, diagram number 5 and diagram number 6.

b) Copy and complete the table below:

Diagram number	Number of dots
1	6
2	10
3	
4	
5	
6	

c) Without drawing the diagrams, state the number of dots in:

i) diagram number 10

ii) diagram number 15.

d) If you write x for the diagram number and n for the number of dots, write down a formula involving x and n.

8. Solve the equations.

a) $x - 6 = 3$ **b)** $x + 9 = 20$

c) $x - 5 = -2$ **d)** $3x + 1 = 22$

9. Solve the equations.

a) $3x - 1 = 20$ **b)** $4x + 3 = 4$

c) $5x - 7 = -3$

10. Nadia said: 'I thought of a number, multiplied it by 6, then added 15. My answer was less than 200'.

a) Write down Nadia's statement in symbols, using x as the starting number.

b) Nadia actually thought of a prime number. What was the largest prime number she could have thought of?

11. Solve the equations for x.

a) $x^3 = 8$ **b)** $3^x = 9$ **c)** $2^x = 16$

12. Simplify:

a) $(x^2)^4$ **b)** $(n^3)^3$ **c)** $4a^2 \times 3a$

Examination-style exercise 2

NON-CALCULATOR

1. Look at the sequence of numbers 9, 13, 17, 21, …

a) Write down the next number in the sequence. [1]

b) Find the 10th number in the sequence. [1]

c) Write an expression, in terms of n, for the nth number in the sequence. [1]

d) Babar says that 84 is a term in the sequence.
Is he correct? Explain your answer. [1]

2. a) The first four terms of a sequence are 11, 6, 1, −4.

i) Write down the next two terms of the sequence. [2]

ii) State the term-to-term rule for the sequence. [1]

iii) Write down an expression for the nth term of this sequence. [2]

b) The first four terms of another sequence are −4, 1, 6, 11.
Write down an expression for the nth term of this sequence. [2]

c) Add together the expressions for the nth terms of both sequences.
Write your answer as simply as possible. [1]

3. Look at this sequence: 3, 8, 15, 24, …

a) Write down the next term. [1]

b) Write down the 10th term. [1]

c) Write down an expression for the nth term of the sequence. [2]

4. Factorise fully $4xy - 2x$. [1]

5. Expand the brackets and simplify $6x(x - y) + 3x^2$. [2]

6. a) Expand and simplify $4(3c - 4d) - 6c$. [2]

b) Factorise $ab - a^2$. [1]

7. Solve the equations.

a) $3x - 2 = 16$ [2]

b) $\frac{y + 1}{4} = 3$ [2]

c) $2(3z - 7) - 3(z - 4) = -7$ [3]

8. Gracie and Edith are each given x dollars.

a) Edith spends 5 dollars. Write down an expression in terms of x for the number of dollars she has now. [1]

b) Gracie doubles her money by working and then is given another 7 dollars.
Write down an expression in terms of x for the number of dollars she has now. [1]

c) Gracie now has four times as much money as Edith. Write down an equation in x to show this. [1]

d) Solve the equation to find the value of x. [3]

9. Solve the equation $\frac{x}{4} + 7 = 12$. [2]

10. Solve the equation $4 - 5x = 2x + 7$. [2]

11. Write down the value of x when:

a) $3^x = 9$ [1]

b) $2^x = \frac{1}{16}$ [1]

12. Simplify:

a) p^0 [1]

b) $(x^3)^4$ [1]

13. Simplify:

a) $\left(\frac{1}{r}\right)^0$ [1]

b) $p^3 \times p^6$ [1]

c) $(x^3)^{-4}$ [1]

14. Three of the following five statements are correct.

A $0.06066 \leqslant 0.06606$

B $0.06066 \neq 0.06606$

C $0.06066 = 0.06606$

D $0.06066 < 0.06606$

E $0.06066 > 0.06606$

Write down the letters that correspond to the three correct statements. [2]

15. x is an integer.

Write down all the values of x that satisfy the inequality $-7 < 2x \leqslant 8$. [2]

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